Section 4.2 (Area)

Before covering the idea of area under a curve, let's introduce / review sigma notation…

$\sum_{i=1}^{n} a_i$ = $a_1 + a_2 + a_3 + \ldots + a_n$ --- Sum up all terms a_1 to a_n

Examples:

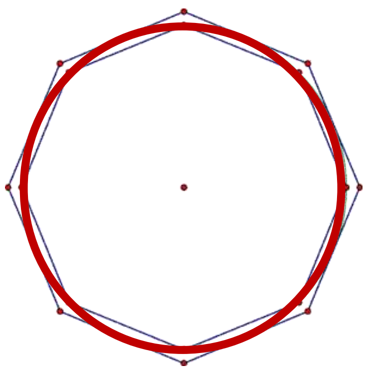
$\sum_{i=1}^{4} i$ = 1 + 2 + 3 + 4 = 10

$\sum_{i=0}^{4}(i-1)$ =

$\sum_{j=2}^{5} 2^j$ =

$\sum_{i=1}^{4} \frac{1}{i}(4)$ =

The area of polygons can usually be found using rectangles and triangles, but the area under curves might involve inscribed and circumscribed polygons. Consider the area of the circle below (bounded by the given polygons)



A similar technique can be used to estimate the area of a plane region.

Example: Sketch on the board 4 inscribed and 4 circumscribed rectangles to bound the area under the given curve $f(x) = -x^2 + 5$ between x = 0 and x = 2

The width of each rectangle is ________, and the height can be estimated by f(x) at the given endpoint.

Reasonably, this area would be better bounded by more rectangles.
(if time, consider doubling the rectangles and using midpoints)

Finding the area under the curve involves finding the sums of the areas of these rectangles (circumscribed and inscribed) and bounding our curve. Note that the width of n rectangles between x=a and x=b is given by $\Delta x = (b-a)/n$

Example: Find the width of each rectangle in the above example if using n=4 rectangles versus n=8 rectangles.

The area of the inscribed rectangles (A=lw) is called a lower sum and is evaluated as $s(n) = \sum_{i=1}^{n} f(m_i)\,\Delta x$

The area of the circumscribed rectangles is called a upper sum and is evaluated as $S(n) = \sum_{i=1}^{n} f(M_i)\,\Delta x$

Review example 4 in the book and consider $\lim_{n\to\infty} s(n)$=

Let f be continuous and nonnegative on the interval [a,b]. The area of the region bounded by the x-axis and vertical lines x = a and x = b is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \text{ where } x_{i-1} \le c_i \le x_i \text{ and } \Delta x = (b-a)/n$$

Example: Find the area of the region bounded by the graph of $f(x) = x^2$, the x-axis, and the vertical lines x = 0 and x = 2.

Example: Find the area of the region bounded by the graph of $f(x) = 9 - x^2$, the x-axis, and the vertical lines x = 1 and x = 3

Homework problems: 1,2,7,13,15,21,25,33,45,47,55 (required) 2,10,26,34,46,57,61 (recommended)