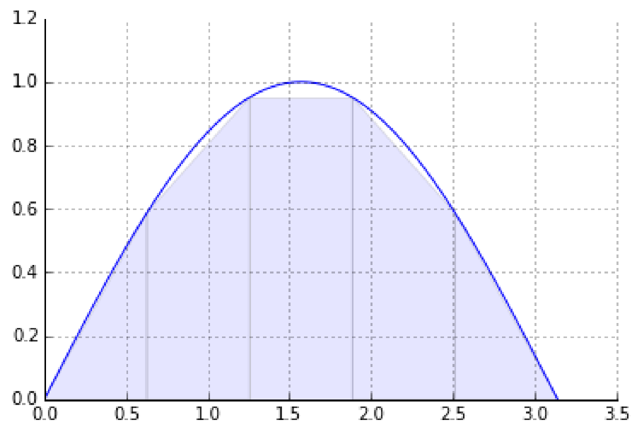
Section 4.6 (Numerical Integration)

Some elementary functions do not have basic antiderivatives (consider $\sin(x^2)$ or $\sqrt{x}\cos(x)$), so we use numerical approximation techniques. One such techniques is the trapezoidal rule.

The **trapezoidal rule** for approximating $\int_a^b f(x)dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)]$

Observe coefficients 1,2,2,…2,1

Example: Use the trapezoidal rule to approximate $\int_0^\pi \sin(x)dx$ with n = 5 and n = 10 trapezoids.



In addition to using rectangles and trapezoids, we can also use a quadratic (polynomial) approximation (curve). The details behind this approach involve finding a general formula for integrating a quadratic…

$$\int_a^b (Ax^2 + Bx + C)dx = \frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx\,\Big]_a^b = \quad \ldots \quad = \left(\frac{b-a}{6}\right)\left[p(a) + 4p\left(\frac{a+b}{2}\right) + p(b)\right]$$

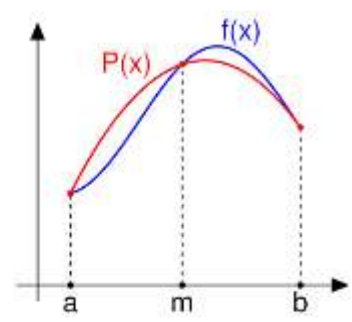
See book for more details (this formula is used in the proving of Simpson's Rule)

Note that the quadratic subintervals need 3 points in x to estimate with Simpson's rule

Simpson's rule for approximating $\int_a^b f(x)dx \approx \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 4f(x_{n-1}) + f(x_n)]$

Observe coefficients 1,4,2,4,…2,4,1

Example: Use Simpson's Rule to approximate $\int_{-\pi/2}^{\pi/2} \cos(x)dx$ for n = 4 and n = 8



Review error estimation and complete homework exercises as time permits

Homework problems: 1,3,7,23 (required) 41 (recommended)